Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2013

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3A/3B

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

Number of additional answer booklets used (if applicable): ☐

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 13-091

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33⅓
Section Two: Calculator-assumed	13	13	100	100	66⅔
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **(100 Marks)**

This section has **13** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 8 **(6 marks)**

Given that $\log_a 4 = x$ and $\log_a 5 = y$,

(a) write expressions, in terms of x and y, for:

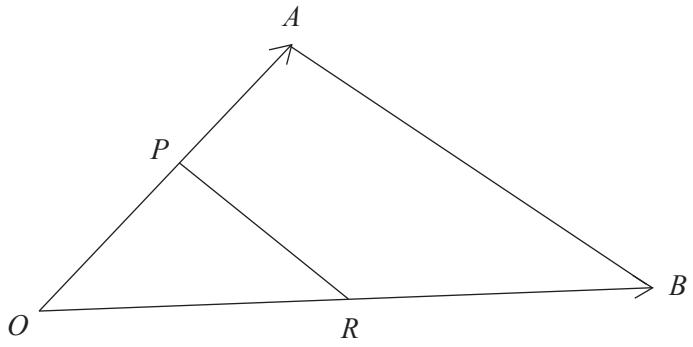
(i) $\log_a 0.8$. (2 marks)

(ii) $\log_a 100$. (2 marks)

(b) Evaluate exactly a^{3x}. (2 marks)

Question 9 **(6 marks)**

In the diagram below, point O is the origin, with points A and B having respective position vectors as $\boldsymbol{a}$ and $\boldsymbol{b}$. Points P and R are the respective midpoints of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.



Write vector expressions, in terms of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, for:

(a) $\overrightarrow{OR}$ (1 mark)

(b) $\overrightarrow{AB}$ (1 mark)

(c) $\overrightarrow{PR}$ (2 marks)

(d) Comparing your expressions for vectors $\overrightarrow{AB}$ and $\overrightarrow{PR}$, deduce two geometric properties regarding $\overrightarrow{AB}$ and $\overrightarrow{PR}$.

(2 marks)

Property one:

Property two:

See next page

Question 10 **(6 marks)**

(a) Given functions f, g and h such that $h(x) = (f \circ g)(x)$ where:

$$f(x) = 2x^2 + 1$$
$$g(x) = \sqrt{x}$$

Determine:

(i) the defining rule for $h(x)$. (2 marks)

(ii) the domain for $h(x)$. (1 mark)

(b) A sequence is given by $\left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \left(\frac{1}{2}\right)^4, \ldots$ Which term will be the first to produce a value less than 10^{-7}? (3 marks)

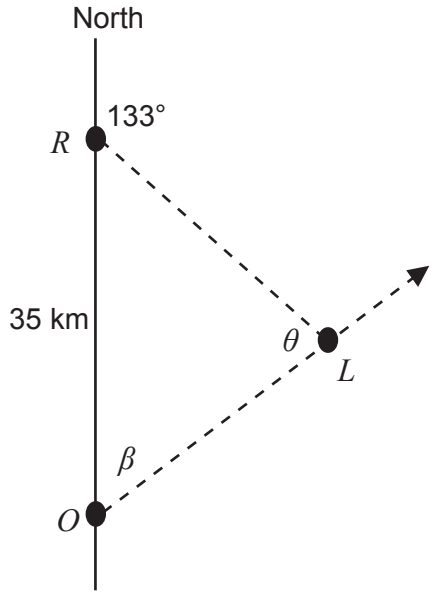
See next page

Question 11 **(9 marks)**

(a) A sector of a circle with radius 20 cm has an exact area of 40π cm^2. Determine the exact perimeter of this sector. (4 marks)

See next page

(b) A light aircraft departs from an airport at point O at a constant speed of 180 km/h and heads in a particular direction. A radar operator at point R, 35 km north of point O, observes the light aircraft from its take-off and notes that, 10 minutes after take-off, it is at a bearing of 133°.



(i) Determine how far the aircraft has travelled in the 10 minutes of its flight. (1 mark)

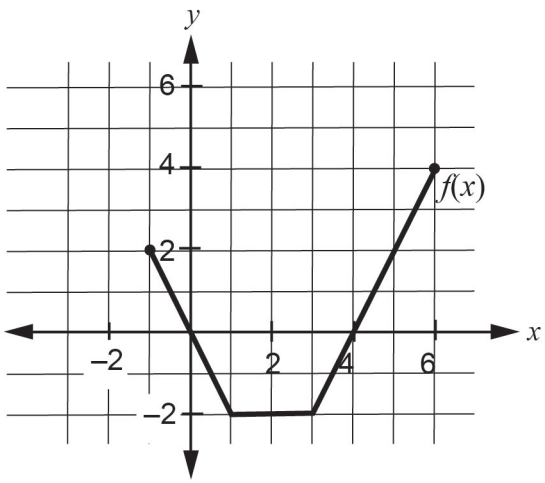
(ii) Calculate the value(s) for θ, correct to the nearest degree. (3 marks)

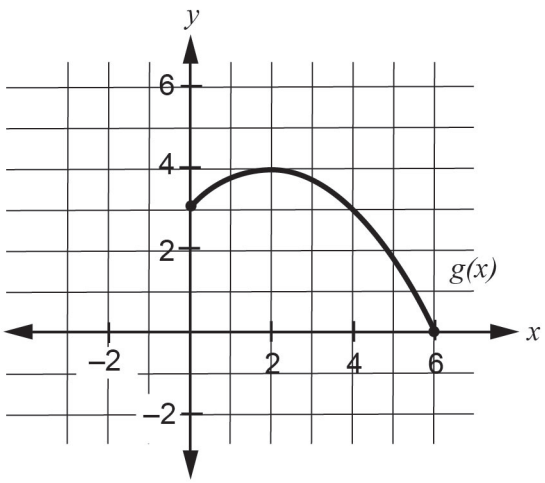
(iii) Hence, determine the possible bearing(s) for the direction of the aircraft, correct to the nearest degree. (1 mark)

See next page

Question 12 **(13 marks)**

The graphs of functions $y = f(x)$, $-1 \le x \le 6$ and $y = g(x)$ are shown below. The defining rule for $y = g(x)$ is $g(x) = -0.25(x-2)^2 + 4, \ 0 \le x \le 6$.





(a) State the range of $y = f(x)$. (1 mark)

(b) Evaluate:

(i) $(f \circ g)(2)$. (2 marks)

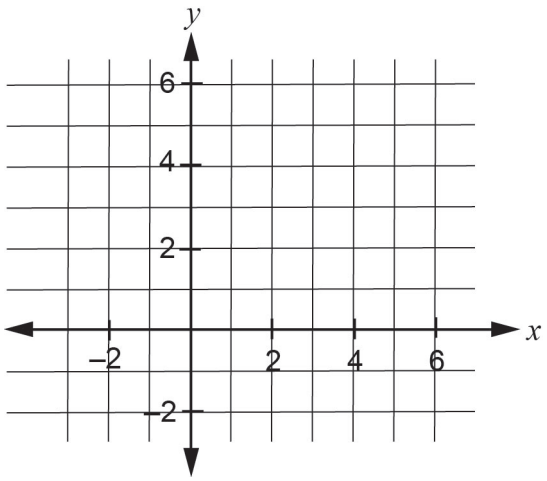
(ii) $(g \circ f)(3)$. (2 marks)

The function composition defined by $(g \circ f)(x)$ is not defined for all real values of x.

(c) State the domain for which $y = (g \circ f)(x)$ is defined. (2 marks)

See next page

(d) Sketch the graph of $y = f'(x)$, the derivative of function f, on the axes below. (3 marks)

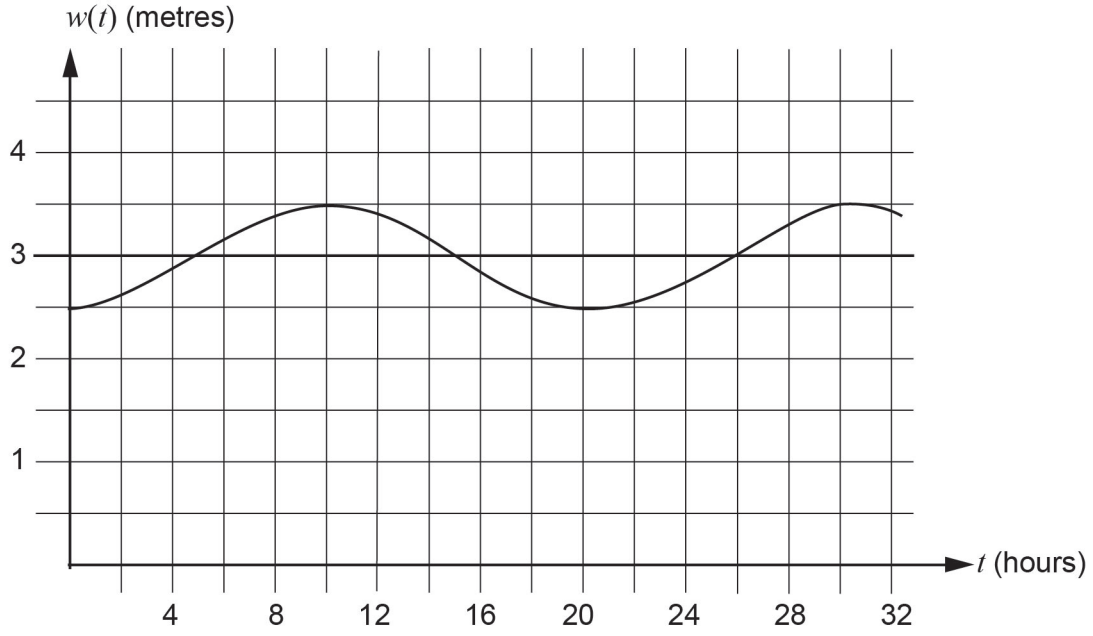


(e) Solve the equation $f(x) = g(x)$, giving solution(s) in exact form. (3 marks)

Question 13 **(7 marks)**

Data on the variation of the height of the water level $w(t)$, measured in metres, at Betty's Jetty was gathered for t hours after midnight on December 21, 2012.

The graph of this variation is shown below:



The graph shows initially that the water level was at low tide. The trigonometric function

$$w(t) = a\cos(bt) + c$$

is used to model the variation in the water level.

(a) Explain, with reasoning, why $a = -0.5$, $b = \dfrac{\pi}{10}$ and $c = 3$. (3 marks)

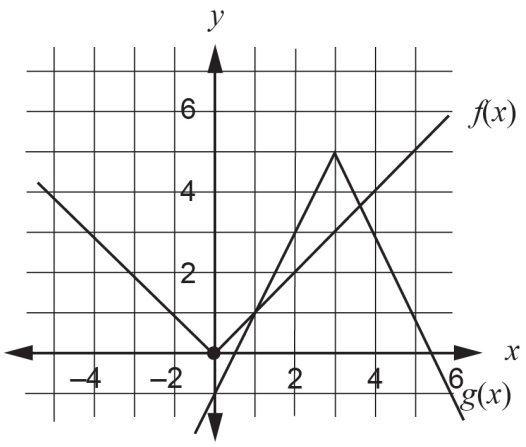
(b) At what time on December 21 was the water level at high tide? (1 mark)

(c) For what percentage of the day, on December 21, was the water level above a height of 3.2 metres? (3 marks)

See next page

Question 14 **(7 marks)**

(a) The graphs of $f(x) = |x|$ and $g(x) = -2|x-3| + 5$ are shown below:



(i) Solve the inequality $|x| < -2|x-3| + 5$. (2 marks)

(ii) Given that the equation $|x| = a|x-3| + b$ has solution set $0 \leq x \leq 3$, determine, with appropriate reasoning, the values for the constants a and b.

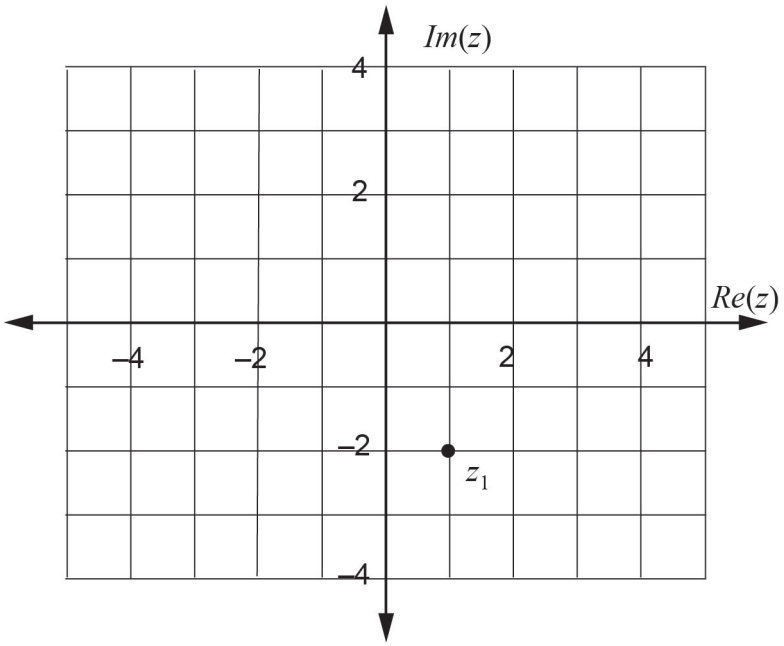
(3 marks)

(b) Solve for x in terms of k: $|2x-k| = 8$. (2 marks)

See next page

Question 15 **(4 marks)**

The complex number $z_1 = 1 - 2i$ is represented on the Argand diagram shown below.



(a) If $z_2 = z_1^2$, show that $z_2 = -3 - 4i$. (1 mark)

(b) Plot the complex number $z_3 = \dfrac{\overline{z}_2}{z_1 + i}$ on the Argand diagram above. (3 marks)

See next page

Question 16 **(6 marks)**

Vectors $\boldsymbol{a}$, $\boldsymbol{b}$ are defined as $\boldsymbol{a} = 3\boldsymbol{i} + 4\boldsymbol{j}$, $\boldsymbol{b} = \boldsymbol{i} + 4\boldsymbol{j}$.

(a) Determine $2\boldsymbol{a} + \boldsymbol{b}$. (1 mark)

(b) Show that $|2\boldsymbol{a} + \boldsymbol{b}| < |2\boldsymbol{a}| + |\boldsymbol{b}|$. (3 marks)

(c) Determine any vector $\boldsymbol{c}$ such that $|\boldsymbol{b} + \boldsymbol{c}| = |\boldsymbol{b}| + |\boldsymbol{c}|$.
Explain your choice for vector $\boldsymbol{c}$. (2 marks)

Question 17 **(5 marks)**

Consider the pattern below:

n	*T(n)*	Exact value
1	$1+\frac{1}{1}$	2
2	$\left(1+\frac{1}{2}\right)^2$	$\frac{9}{4}$
3	$\left(1+\frac{1}{3}\right)^3$	$\frac{64}{27}$
4	$\left(1+\frac{1}{4}\right)^4$	$\frac{625}{256}$

(a) Determine the exact value of $T(5)$. (2 marks)

(b) Write a formula for the $T(n)$, where n is any positive integer. (2 marks)

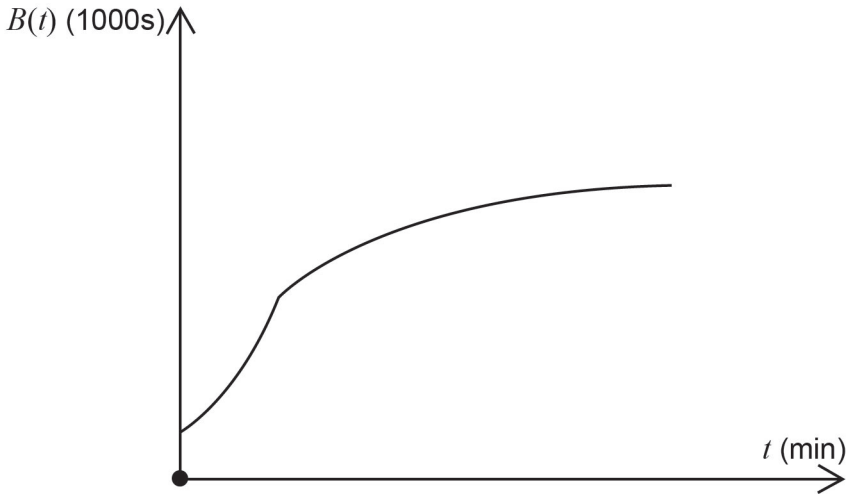
(c) Determine the exact limiting value of $T(n)$ as $n \to \infty$. (1 mark)

See next page

Question 18 **(10 marks)**

A synthetically created bacteria colony called Mathematicus-Specialus was allowed to grow exponentially for 20 minutes. At the end of the 20 minutes of growth, there were 60 000 of the bacteria.

The bacteria colony was then treated with a chemical that inhibited its growth as shown in the graph below:



The growth of the bacteria is modelled by the piece-wise function $B(t)$, where:

$B(t)$ = the number of bacteria in 1000s after t minutes

$$B(t) = \begin{cases} B_0\, 2^{0.1t} & , \quad 0 \leq t \leq 20 \\ 100 - 80(2^{-kt}) & , \quad t > 20 \end{cases}$$

(a) Explain what feature of the graph provides evidence that the growth of the bacteria was inhibited. (1 mark)

(b) State why $B(20) = 60$. (1 mark)

(c) Determine the value of B_0. (3 marks)

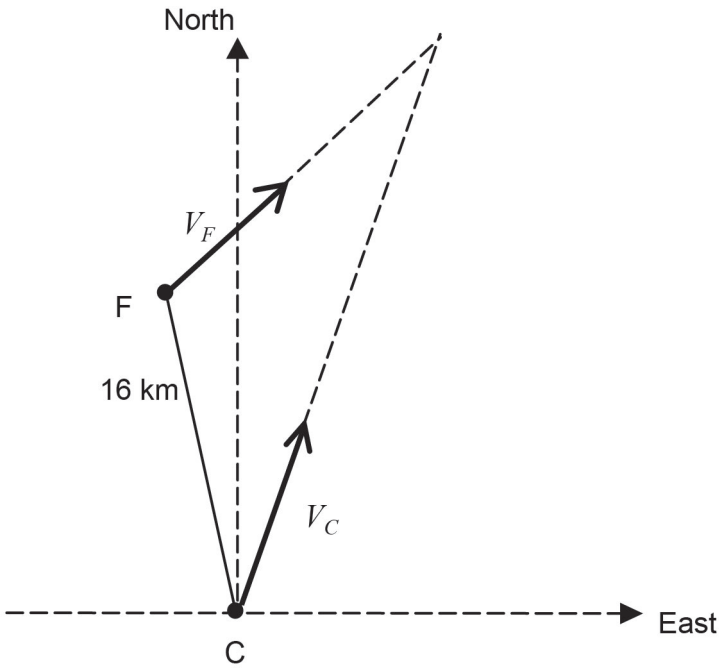
(d) Determine the value of the constant k. (3 marks)

(e) Provide appropriate mathematical reasoning to show that the number of bacteria in the colony stabilised in the long run, and state this number. (2 marks)

See next page

Question 19 **(14 marks)**

A coast guard vessel, at C, is ordered to intercept a fishing boat, F, that is 16 km away and on a bearing of N10° W. The fishing boat is moving at 28 km/h on a course of N 55°E.



Assuming that the coast guard vessel is initially situated at the origin, then:

(a) for the initial positions, show that $\overrightarrow{CF} = \begin{pmatrix} -2.778 \\ 15.757 \end{pmatrix}$ (correct to 0.001 km). (2 marks)

(b) show that the velocity of the fishing boat $\boldsymbol{v}_F = \begin{pmatrix} 22.936 \\ 16.060 \end{pmatrix}$ (correct to 0.001 km/h). (2 marks)

Suppose that the velocity of the coast guard vessel, $\boldsymbol{v}_C$, is expressed as $\boldsymbol{v}_C = (a,b)$.

(c) Given that the coast guard vessel travels at a speed of 32 km/h, write an equation relating a and b. (1 mark)

See next page

(d) Write the expression for the velocity of the coast guard vessel relative to the fishing boat, denoted as $_C\boldsymbol{v}_F$. (1 mark)

(e) Using the relative velocity $_C\boldsymbol{v}_F$, from part (d), write a mathematical equation that will represent the coast guard vessel intercepting the fishing boat. (2 marks)

(f) Determine the time it takes in hours (correct to the nearest 0.01 hours) for the coast guard vessel to intercept the fishing boat. (4 marks)

(g) How far did the coast guard vessel have to travel to intercept the fishing boat, correct to the nearest kilometre? (2 marks)

See next page

Question 20 **(7 marks)**

The quadratic equation $z^2 - 4z + c = 0$ will, for any real number c, have two complex solutions z_1 and z_2. Since z_1 and z_2 are solutions, then the equation can be written in the form:

$$(z - z_1)(z - z_2) = 0.$$

(a) For $c = 5$, show that $z_2 = \overline{z}_1$. (3 marks)

See next page

In part (a), the value of c was given: $c = 5$.

Now consider d to be a particular real number. Assume that $z^2 - 4z + d = 0$ has complex solutions $z = z_1$ and $z = z_2$.

(b) Write the standard quadratic equation, in terms of d, that has solutions $z = 3z_1$ and $z = 3z_2$.
(4 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

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